PRELIMINARY VERSION
The Small Price Effect in Trading Prices: an Experimental Study *

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Abstract

Studies in neuropsychology show that the human brain processes small numbers and large numbers differently. Small numbers are processed on a linear scale, while large numbers are processed on a logarithmic scale. Recent papers in finance and accounting also indicate that market participants exhibit a bias in their perception of the future return distribution of small and large price stocks. In this paper , we report the results of an experiment in which we test for the existence of a small price effect. Our experiment consists in 8 sessions of two sequential experimental markets where the cash-flows and endowments are 12 times higher in one market compared to the other. We find that optimism, measured as the relative difference between transaction prices and fundamental values, is significantly greater in small price markets. This result is obtained both within and across subjects. Our experimental results indicate that price level influences the way people perceive relative price variations (*i.e.*, returns), a result at odds with standard finance theory but consistent with: 1) a number of empirical observations on financial markets; and, 2) the use of different mental scales for small and large prices.

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1 Introduction

Normative decision theory assumes that expectations are not sensitive to changes in the way information is presented. For instance, the magnitude of a stock price should not influence return expectations, future realized returns nor portfolio choices. Empirical evidence on financial markets, however, show that the stock price level has an impact on stock returns, analysts' forecasts and investors' portfolio choice. Schultz (2000) finds that 1) retail investors hold lower-priced stocks than institutions, and 2) the number of retail investors among shareholders of a firm increases after a forward stock split that decreases the stock price without changing the fundamentals of the firm. Green and Hwang (2009) show that the returns of small (large) price stocks comove more together than with the returns of large (small) price stocks. Birru and Wang (2016) provide evidence that investors overestimate the room to grow for small price stocks, compared to large price stocks. Baker, Greenwood, and Wurgler (2009) find that firms manage nominal prices through forward stock splits when investors are willing to pay a premium for small price stocks.

Roger, Roger, and Schatt (2016) show that financial analysts exhibit a small price bias when issuing price forecasts (target prices). Analysts exhibit a greater optimism for small price stocks compared to large prices stocks. Target price implied returns are greater for small price stocks and this difference cannot be explained by risk factors. The authors link their findings to recent research in neuropsychology devoted to the mental representation of numbers (Dehaene, 2011, for a review). The main model for number representation in the human brain is the Weber-Fechner law (Nieder, 2005). This law states that the brain uses a logarithmic scale to represent numbers: increasingly larger numbers are subjectively closer together. This theoretical framework is in line with the assumption of standard finance theory stating that people select stocks on the distribution of future returns (logarithmic scale), not price expectations (linear scale). In other words, under Weber-Fechner law, returns implied by price forecasts should be independent of the stock price level. However, recent papers (Dehaene, Izard, Spelke, and Pica, 2008; Hyde and Spelke, 2009) have shown that the Weber-Fechner law is not satisfied for small numbers. People tend to use a linear scale for small numbers and they compress large numbers on a logarithmic scale (Viarouge, Hubbard, Dehaene, and Sackur, 2010). The use of different scales for small and large numbers is likely to impact market participants' expectations.

In this paper, we report the results of an experiment in which we test for the existence of a small price effect. Our experiment consists in 8 sessions of two sequential experimental markets where the cash-flows and endowments are 12 times higher in one market compared to the other. In four sessions, the first experimental market is the "small price" market and the following market is the "large price" market. In the four other sessions, the order of the two successive markets is reversed. Overall, we find that these two different experimental markets generate different trading price processes. Consistent with the linear vs. logarithmic scales in processing numbers, we find that subjects' optimism is greater in "small price" markets. This result is obtained both within and across subjects.

2 Sketch of the experimental design

In this paper, we aim at controlling all risk factors to focus on the potential effect of price magnitude in the determination of returns. It is known since Smith, Suchanek, and Williams (1988) that experimental markets lead to bubbles followed by crashes that drive prices to fundamental values at the end of the market. As a consequence, the assumption of different types of scales for small and large numbers should lead to larger bubbles and sharper crashes in small-price markets. We run 8 sessions with 2 successive experimental markets per session with the same traders in the two markets of a given session. One market is a small price market and the other is a large price market. In each market of a given session, our protocol follows the spirit of Smith, Suchanek, and Williams (1988) (SSW) but departs from the SSW conditions in several ways. Being given the purpose of our paper, we first want to avoid inducing anchors in the traders' minds by choosing a constant fundamental value. Second, prices in a given experimental market should keep the same magnitude. It therefore excludes the initial SSW design where fundamental values converge to 0 over time. As a consequence, we design a sequence of random cash-flows whose distribution is known by subjects. The cumulated payoff received by an investor holding the stock at the end of the market is the sum of the cash-flows drawn at random at each date t, $t = 1, ...10$. No intermediate dividends are paid to the participants (see the Appendix for details).

Our design has several advantages beyond the stability of the price magnitude in a given market. The stochastic process of fundamental values is a martingale with respect to the filtration generated by the cash-flow process. As a consequence, it is easy to define a simple measure of optimism using the relative deviation of trading prices with respect to the fundamental value. The second advantage is that we can build sessions with two successive markets, one with small prices and the other with large prices, in such a way that the distribution of large cash-flows (and consequently of fundamental values) is 12 times the distribution of small cash-flows.

The experiment was conducted at LEEM, the computerized laboratory of the University of Montpellier I, with the software z-Tree (Fischbacher, 2007). The 8 sessions involved 9 traders each. The 72 subjects were randomly selected from a pool of student-subjects containing more than 5000 volunteers from the Universities of Montpellier. In each session, groups of 9 anonymous participants were randomly formed and remained fixed for the whole session. The experiment consisted in 20 periods divided into 2 independent markets lasting $T = 10$ periods each. 4 sessions were of the SL (LS) type, starting by a small (large) price market followed by a large (small) price market. This design allows comparison between and within subjects to test whether small and large numbers are processed differently by participants.

Subjects were provided with incentives in a usual way. They first completed a task giving them the possibility of earning €30. Then, one of the two markets (small-price or large-price) was drawn at random to be paid. The aggregate amount of 9 × €30 = €270 was shared among subjects in proportion of their final wealth at the end of the market randomly drawn to be paid. Table 1 shows the composition of portfolios and the sequences of cash-flows. In a given market, the initial theoretical value of all portfolios is equal. P1 to P3 (P4 to P6) are the portfolios in the small (large) price market. The value of the theoretical portfolio in the large-price market is worth 12 times the value of the theoretical portfolio value in the small price market. Panel B shows the two sequences of cash-flows in the small (large)-price market. The vector of cash-flows in the large-price market is 12 times the vector of cash-flows in the small-price market. A permutation has, however, been used to avoid that subjects recognize the cash-flow sequence. These cash-flow sequences imply an initial fundamental value of 6 (72) for the small (large) price market.

3 Preliminary results

Let $V^S = V_t^S, t = 0, ..., T$ ($V^L = V_t^L, t = 0, ..., T$) the fundamental value process on the small (large) price market and $P^S = P_t^S, t = 1, ..., T$ ($P^L = P_t^L, t = 0, ..., T$) the median prices observed on the experimental market. To make these quantities directly comparable, we denote $O^S = O_t^S, t = 1, ..., T$ ($O^L = O_t^L, t = 1, ..., T$) what we call the average market optimism on the small (large) price market. O_t^S is defined by:

$$O_t^S = \frac{P_t^S - V_{t-1}^S}{V_{t-1}^S} \tag{1}$$

and O_t^L is defined in the same way.

Table 1
Composition of portfolios

Panel A: Portfolio composition						
	P1	P2	P3	P4	P5	P6
Number of stocks	3	6	9	3	6	9
Amount of cash	82	64	46	984	768	552

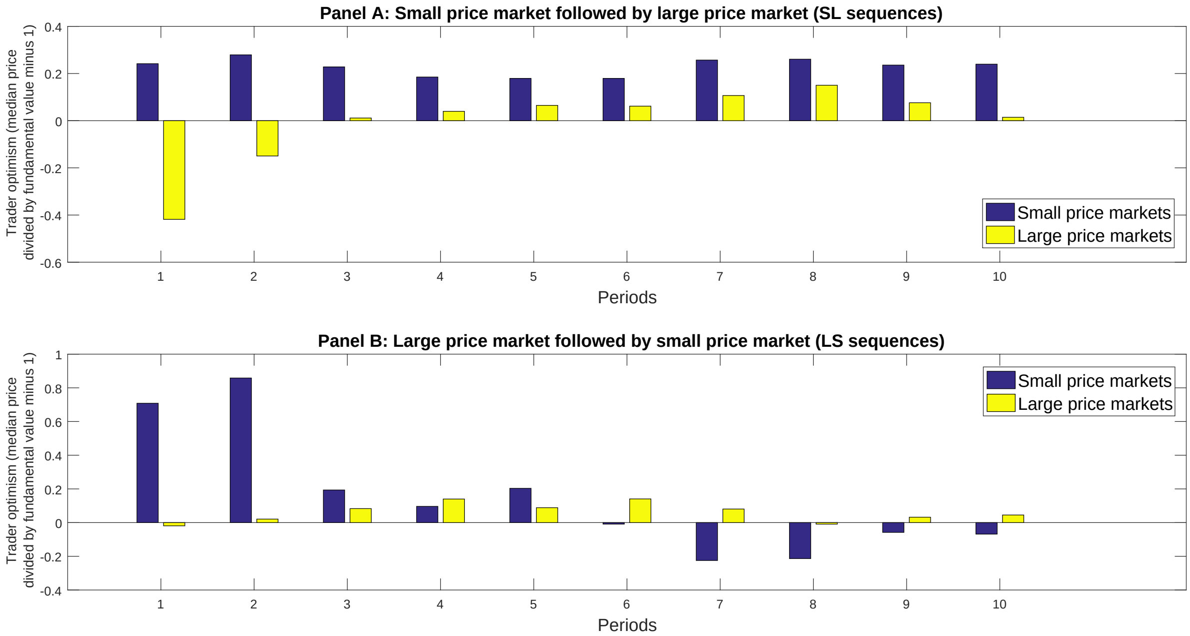
Panel B: Time series of cash-flows										
Periods	1	2	3	4	5	6	7	8	9	10
Small price market	0.6	0.3	0.6	0.9	0.6	1.2	0.9	0.3	0.0	0.6
Large price market	10.8	7.2	7.2	7.2	7.2	14.4	10.8	0.0	3.6	3.6

The first (second) line of Panel A gives the quantities of stocks (cash) in portfolios. Portfolios P1 to P3 (P4 to P6) correspond to the small (large) price market. Quantities are determined to have a theoretical portfolio value in the large price market equal to 12 times the theoretical portfolio value in the small price market. The first (second) line of Panel B gives the sequence of cash-flows in the small (large) price market. The sequence of cash flows in the large price market is 12 times the sequence of cash flows in the small price market. A permutation has, however, been used to avoid that subjects recognize the sequence.

The cash-flow process and the way it is revealed over time implies that, in a world of rational traders, trades should be justified at the beginning of the market by differences in risk aversion. The more risk averse subjects should sell their stocks to the less risk averse. Trade prices should be lower than the fundamental value of the stock. Prices above the fundamental value can only occur if some subjects are risk lovers. In all cases, there is no reason to observe significant differences in optimism measures on the two markets with the same traders.

Over 8 sessions lasting 10 periods, there are 160 measures of optimism, 80 for small price markets and 80 for large price markets. Figure 1 reports the optimism measures for the SL (Panel A) and LS sequences Panel B). In panel A, the blue (yellow) bars show the average of O_t^S (O_t^L) over the first (second) market of the four SL sessions. Conversely, in panel B, the blue (yellow) bars show the average of O_t^S (O_t^L) over the second (first) market of the four LS sessions. We find that subjects exhibit greater optimism in small price markets compared to large price markets. This results is particularly striking for the SL sequence. The measures of optimism are, however, different, depending on the position of the market in the SL or the LS sequence. In particular, there is a restart effect at the beginning of the second market. When the first market is a small price market, it is likely that one or several trades in the first periods of the second market are realized at a small price. It happens if one (or several) subject does not pay sufficient attention to the change of the cash-flow process. It also happens the other way around when the first market is a

Figure 1
Optimism in sequential markets



large price market. Some trades in the first periods of the second (small price) market are likely to occur at a large price. In the two situations, the apparent differential optimism between the two price magnitudes is in favor of the small price case.

The cleanest data to compare the optimism measures in the two types of market come from the first of the two sequential markets SL and LS. Figure 1 shows the detailed results of the first market of the 8 sessions. The blue (yellow) bars show the average of O_t^S (O_t^L) over the four SL (LS) sessions. The average optimism in SL sessions is approximately 20%. It is stable over time and the premium with respect to the fundamental value remains (on average) until the end of the market. It should be noted, however, that some "irrational" prices were observed in the last periods. A trade price is said irrational when it is above (below) the maximum (minimum) possible final payoff. For example, if the sum of the first 8 realized cash-flows is 5.1, the maximum cumulated cash-flow until the end of the market is $5.1 + 2.4 = 7.5$ because of the cash-flow distribution given in Table 1. The story is different for large price markets. The shape of the bar chart is more conventional. The bubble develops over the first half of the market and decreases in the second half to become negligible in the end.

Table 2 shows the statistical comparison of optimism variables. Panel A provides the test corresponding to the data in Table 2. Panel B compares optimism in SL sessions. The line “Small prices” gives the median value of optimism calculated with the aggregate set of small prices in the four SL sessions. The line “Large prices” gives the same information for large prices (occurring in second markets). The difference appearing in the third line is always positive and highly significant. The restart effect referred to above appears in the first period of the second market. The median large price is 30% lower than the fundamental value. Even beyond the first periods the difference remains significant and no bubble is observed in the second market with large prices. Panel C shows the restart effect mainly in periods 1 and 2 where the optimism in small prices is 67% and 35% respectively. An interesting observation is the overreaction in the following periods. In particular, the second half of the small price market shows large negative optimism (or excessive pessimism).

Figure 2
Aggregate optimism in first markets

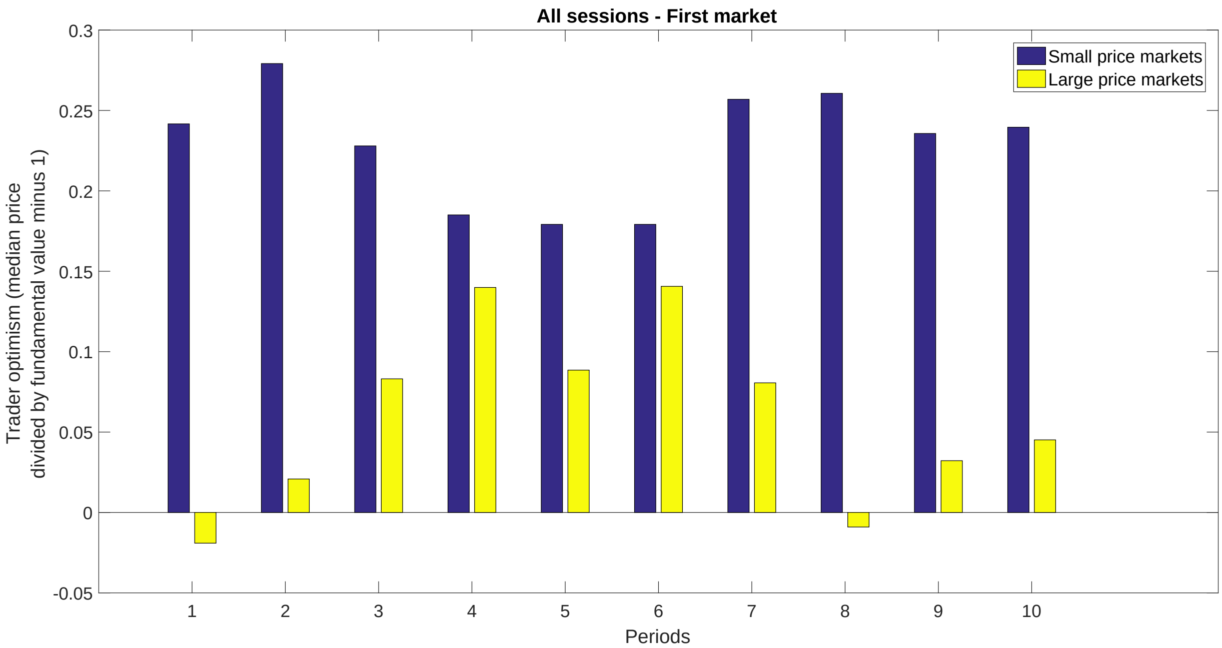


Table 2
Median optimism (median trading price divided by fundamental value minus 1)

Panel A: First markets										
Periods	1	2	3	4	5	6	7	8	9	10
Small prices	0.1167	0.1667	0.2281	0.2063	0.1250	0.1500	0.1852	0.0145	0.0758	0.0833
Large prices	0.1111	0.1111	0.0965	0.1696	0.0417	0.2083	0.0101	−0.0097	0.0543	0.0625
Difference	0.0056**	0.0556***	0.1316***	0.0368	0.0833**	−0.0583	0.1751***	0.0242**	0.0215***	0.0208

Panel B: Small price market to Large price markets										
Periods	1	2	3	4	5	6	7	8	9	10
Small prices	0.1167	0.1667	0.2281	0.2063	0.1250	0.1500	0.1852	0.0145	0.0758	0.0833
Large prices	−0.3056	−0.1402	−0.1270	−0.0741	−0.0741	−0.0058	−0.0338	−0.0914	0.0031	0.0210
Difference	0.4222***	0.3069***	0.3551***	0.2804***	0.1991***	0.1558***	0.2190***	0.1059***	0.0727***	0.0624***

Panel C: Large price market to Small price markets										
Periods	1	2	3	4	5	6	7	8	9	10
Small prices	0.6667	0.3492	0.1111	0.1032	−0.0476	−0.1984	−0.2029	−0.2431	−0.0909	−0.0476
Large prices	0.1111	0.1111	0.0965	0.1696	0.0417	0.2083	0.0101	−0.0097	0.0543	0.0625
Difference	0.5556***	0.2381***	0.0146	−0.0664	−0.0893	−0.4067***	−0.2130***	−0.2334***	−0.1452***	−0.1101***

Each number in a line ”Small price” or ”Large price” is calculated as follows: for example, the first number 0.1167 in Panel A is the ratio of the median trading price over the first period of the 4 small price markets in SL sessions, divided by the fundamental value minus 1. Statistical significance between the median optimism of small price markets and large price markets is assessed with a Wilcoxon-Mann-Whitney test. ***/**/* represent significance at the 1%, 5% and 10% level.

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4 Appendix

There are 9 traders in each session. Traders are initially endowed with a portfolio of stocks and an amount of experimental money. Each trader i is given a portfolio $\theta_{i,0} = (\theta_{i,0}^0, \theta_{i,0}^1)$, where the superscript 0 identifies the cash endowment (risk-free asset with zero interest rate) and the superscript 1 identifies the risky asset (the stock). $\theta_{i,0}^1$ is the number of units of stock subject i starts with.

The main purpose of the cash-flow process of the risky asset we choose is to keep the magnitude of prices stable during a given market. It is the reason why the risky asset does not pay any dividend before the end of the market. The unique payoff is realized at date $T = 10$ and only the final holders of the stock receive the sum of the cash-flows accumulated since the beginning of the market. Therefore, holding the stock gives the right to receive the cumulated (since date 1) cash-flows at the terminal date. It is equivalent to a firm that pays no dividend and is liquidated at date $T = 10$. The mathematical definition of the stock is a stochastic process of *i.i.d.* random cash-flows $CF = (CF_t, t = 1, ..., T)$.

Typically, the realization of the random cash-flow CF_t is drawn (and publicly released) at the end of each period t. CF_t is drawn in a set of 5 equally likely values. The realization is denoted cf_t. μ and σ are the expectation and the standard deviation of CF_t, respectively. The *i.i.d.* assumption implies that μ and σ do not depend on t. If E_Q denotes the expectation operator under a risk-neutral probability Q, the date-0 theoretical price (fundamental value) of the stock is equal to the sum of the expected future cash-flows

$$V_0 = \sum_{t=1}^{T} E_Q(CF_t) = T\mu \tag{2}$$

The fact that cumulated cash-flows are paid at the end of the market means that, at any date, buying (selling) the stock is equivalent to acquiring (selling) the complete sequence of T cash-flows. The information about cash-flows is progressively revealed to the traders. At each date t, traders observe a realization cf_t of the date-t random cash-flow CF_t. The fundamental value at the beginning of period t is therefore equal to

$$V_t = \sum_{s=1}^{t-1} cf_s + \sum_{u=t}^{T} E_Q(CF_u) \tag{3}$$

In our model, the variations of the fundamental value between two dates $t - 1$ and

t come from the partial resolution of uncertainty at the end of period $t-1$ when the date-$t-1$ cash-flow is revealed. In such a framework, the process of the fundamental value $V = V_t, t = 0, ..., T$ is a martingale with respect to the information generated by the cash-flow process (I_t denotes the information available at date t).

$$V_t = E_Q(V_{t+1} \,|I_t) \tag{4}$$

It turns out that, seen from date 0, the fundamental value has a constant expectation (by the law of iterated expectations). During a market, the conditional expectation of a future fundamental value can be higher or lower than the initial value, depending on the past (and thus already known) sequence of cash-flows. As an example, consider the random return on the risky asset over period t if traders are rational and risk-neutral. If we denote r_t this return, equation 3 implies

$$r_t = \frac{V_{t+1} - V_t}{V_t} = \frac{cf_t - E_Q(CF_t)}{V_t} \tag{5}$$

Equation 5 shows that the one-period expected return $E_Q(r_t \,|I_t)$ is zero, but the conditional variance of the one-period return is not constant over time. In fact, $V_Q(r_t \,|I_t) = \frac{V_Q(CF_{t+1})}{S_t} = \frac{\sigma^2}{V_t^2}$. As a consequence, the variance of the one-period return is lower after a sequence of high cash-flows which increase the stock price, and higher after a sequence of low cash-flows which decrease the stock price.

The *i.i.d.* assumption implies that the variance of the terminal payoff of the stock, $V_T = \sum_{t=1}^{T} CF_t$ is equal to $T \times \sigma^2$. Combined with equation 2, this remark shows that the variance of the gross return $\frac{S_T}{S_0}$ over the entire period is equal to

$$Var_Q(\frac{V_T}{V_0}) = \frac{Var_Q(S_T)}{V_0^2} = \frac{T\sigma^2}{T^2\mu^2} = \frac{1}{T}\frac{\sigma^2}{\mu^2} \tag{6}$$

It appears counter intuitive that the variance of returns decreases with maturity. However, the average price increases linearly with maturity, being given a per period cash-flow probability distribution. This result should not be misinterpreted. It does not mean that the variance of returns increases as time passes. The reason is simple. At date t, the variance of the gross return until date T is written

$$Var_Q(\frac{V_T}{V_t}|I_t) = \frac{Var_Q(V_T|I_t)}{V_t^2} = \frac{(T-t)\sigma^2}{(\sum_{t=1}^{t} cf_t + (T-t)\mu)^2} \tag{7}$$

This quantity is decreasing when t increases most of the time. In fact, when the date-t cash-flow is greater or equal to μ, the denominator of the right hand side of equation 7 increases between $t-1$ and t. Simultaneously the numerator decreases. However, it may happen that this conditional variance increases between two dates when the realized cash-flow is very low. In this situation, the denominator can decrease more than the numerator (which always decreases by σ^2). This special case can occur in the first periods, when the amount of cash-flows already paid is low.

To sum up, everything else equal, the variance of the global return decreases as the maturity of the stock increases[1] but, being given a maturity date, the conditional variance of returns decreases as time passes.

[1]This phenomenon is called time-diversification by finance professionals.